RADIATION FROM TRIANGULAR AND CIRCULAR RESONATORS IN MICROSTRIP

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Abstract

For the fundamental mode, experimental results reveal a slightly higher radiation Q-factor for the triangular microstrip resonator than for the corresponding circular geometry. Theoretical results have been computed for the triangle case. Radiation patterns and interesting circuit applications for the triangle resonator have also been investigated.

Introduction

The radiation characteristics of microstrip resonators are of interest since they limit the unloaded Q-factors which may be obtained, and may necessitate special shielding steps to be taken in the overall subsystem packaging. The two resonator types studied, shown in Fig. 1, can both be used for circulators since each possess the necessary 120° symmetry. For the modes considered there is no field variation in the z-direction, and open-circuit boundaries exist at the resonator periphery. It will be shown that the microstrip resonator of equilateral triangle shape provides slightly lower radiation than the disc shaped unit. Results have been computed from an analysis based on a Hertz vector formulation [1], and good experimental agreement is found for the radiation Q-factor, Q_r [2]. It is also feasible to utilize the triangle resonator as an antenna element.

Microstrip Resonator of Equilateral Triangle Shape

The resonator geometry used is shown in Fig. 2. For the fundamental $TM_{1,0,-1}$ mode, the electric and magnetic field distribution for the resonator are given as [3]:

$$E_z(x,y) = A_{1,0,-1}\left[2\cos\left(\frac{2\pi x}{\sqrt{3}A}+\frac{2\pi}{3}\right)\cos\left(\frac{2\pi y}{3A}\right)+\cos\left(\frac{4\pi y}{3A}\right)\right] \quad (1)$$

$$H_x(x,y) = -jA_{1,0,-1}\,\xi\left[\cos\left(\frac{2\pi x}{\sqrt{3}A}+\frac{2\pi}{3}\right)\cos\left(\frac{2\pi y}{3A}\right)+\sin\left(\frac{4\pi y}{3A}\right)\right] \quad (2)$$

$$H_y(x,y) = -j\sqrt{3}A_{1,0,-1}\,\xi\left[\sin\left(\frac{2\pi x}{\sqrt{3}A}+\frac{2\pi}{3}\right)\cos\left(\frac{2\pi y}{3A}\right)\right] \quad (3)$$

where $A_{1,0,-1}$ is a constant,

$$\text{and } \xi = \sqrt{\frac{\varepsilon_o\varepsilon_r}{\mu_o\mu_r}} = \frac{\sqrt{\varepsilon_r}}{377\Omega} \text{ for } \mu_r = 1 \quad (4)$$

Following an approach similar to that taken by Lewin [1], we have for the total current density the contributions from the conduction currents in the resonator and its image, together with that due to the dielectric polarization:

$$\vec{J} = \vec{J}_{CR} + \vec{J}_{CI} + \vec{J}_P \quad (5)$$

$$= [-H_y(x,y,h) + H_y(x,y,-h)]\vec{n}_x + [H_x(x,y,h) - H_x(x,y,-h)]\vec{n}_y + [jk\,\xi_o(\varepsilon_e - 1)\,E_z(x,y,0)]\vec{n}_z \quad (6)$$

where $k = 2\pi/\lambda_o$, $\xi_o = 1/377\Omega$

$\vec{n}_x$, $\vec{n}_y$ and $\vec{n}_z$ are unit vectors.

For the dielectric polarization component $\vec{J}_p$, the relative permittivity (ε_e) is somewhat arbitrarily taken to be the value of effective permittivity appropriate to uniform microstrip of width equal to that of the triangle at x=0 [6].

The Hertzian vector in free space is [1]

$$\vec{\Pi} = -\frac{j30}{k}\int_V \frac{e^{-jkr}}{r}\vec{J}\,dV \quad (7)$$

$$= \Pi_x\vec{n}_x + \Pi_y\vec{n}_y + \Pi_z\vec{n}_z \quad (8)$$

Using far-field approximations this leads to a calculation of the total radiated power from the resonator into the half-space:

$$P_r = \frac{1}{240\pi}\int_0^{2\pi}\int_0^{\pi/2}\left[|E_\theta|^2 + |E_\phi|^2\right]r_o^2\sin\theta\,d\theta\,d\phi \quad (9)$$

where $E_\theta = k^2(\Pi_x\cos\theta\sin\phi + \Pi_y\cos\theta\cos\phi - \Pi_z\sin\theta)$

$$E_\phi = k^2(\Pi_x\cos\phi - \Pi_y\sin\phi),$$

$$E_r = 0 \quad (10)$$

The stored energy in the resonator may be readily obtained as

$$W = \varepsilon_r\,\varepsilon_o\,A^2\,h\,A_{1,0,-1}^2\,(3\sqrt{3}/16) \quad (11)$$

so that the radiation Q-factor may then be calculated as [2]

$$Q_r = \left(\frac{2\pi\ fW-P_r}{P_r}\right) \qquad (12)$$

$$\%\ \text{Rad. Power} = \frac{2\pi}{Q_r}\ .100\% \qquad (13)$$

A computer program has been written to provide Q_r data for the triangle resonator. Typical CPU time is less than 0.1 sec./point for a Xerox Sigma 9 machine.

Microstrip Disc Resonator

For the disc resonator of Fig 1(a) Morel et al [7] have described a method of analysis somewhat similar to that presented here. Measured data for Q_r has been obtained for the n=1 fundamental mode.

Comparison with Measurement and Applications

The radiation Q-factor, Q_r, has been determined experimentally for both disc-and triangle-shaped resonators from the overall unloaded Q-factors observed with and without a low-loss shielding enclosure [3]. Most of the substrates used were Ti:W-Au coated alumina, with the circuit pattern and jig arranged as illustrated in Fig. 3. The orthogonal type of launcher mounting shown consistently provides lower undesired radiation and surface wave propagation than observed with the conventional in-line mounting. A range of frequencies and substrate thicknesses were employed and the data plotted as in Fig. 4(a). It can be seen that the disc resonator radiates somewhat more than the triangular design for the dominant modes. For the triangular resonator, Fig. 4(b) indicates that better agreement between theory and experiment is obtained for lower values of substrate ε_r. The computed variation of Q_r^{-1} with ε_r is shown in Fig. 5.

The theoretical E- and H-plane radiation patterns are shown in Fig. 6. From these results, together with semi-empirical feed coupling data, linear antenna arrays have been designed. However, unlike the disc or square radiating element, the 120^o symmetry properties of the triangle geometry would appear to preclude the possibility of providing circular polarization.

Other applications for the triangle element may take advantage of the fact that the radiation is somewhat less than for the disc resonator. For example, circulators have been designed with triangular rather than circular junctions and it has been observed that the circulator bandwidth is improved in addition to the marginally better insertion loss. Oscillator circuits have also been investigated in which r.f. power from several diodes is combined in a Kurakawa-type configuration.

Conclusions

The radiation characteristics of disc and triangular shaped resonators in microstrip have been examined. The results are useful to the circuit designer concerned with minimizing losses or spurious coupling between such microstrip resonators when used for circulators, filters and oscillators. This work also contributes much-needed quantitative information to the antenna designer, suggesting new schemes for beam shaping in conformal antennas.

Acknowledgements

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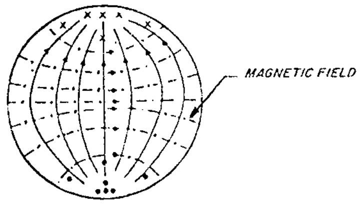
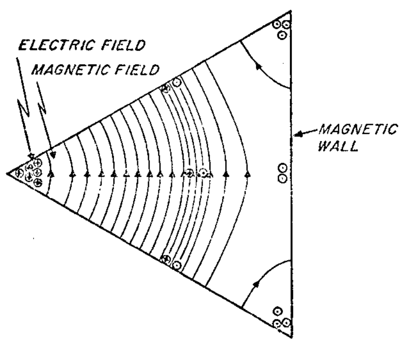


a)



b)

Fig. 1 Dominant Modes in (a) disc [5] and (b) triangular [3] resonators.

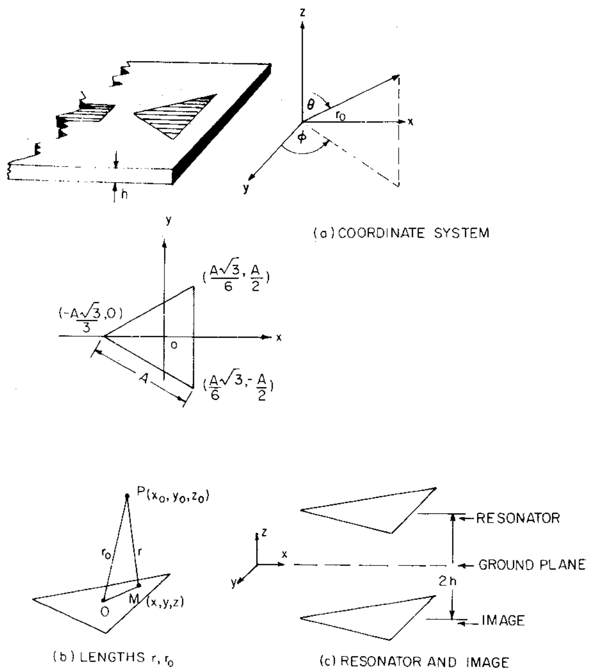


Fig. 2 Triangular resonator geometry.

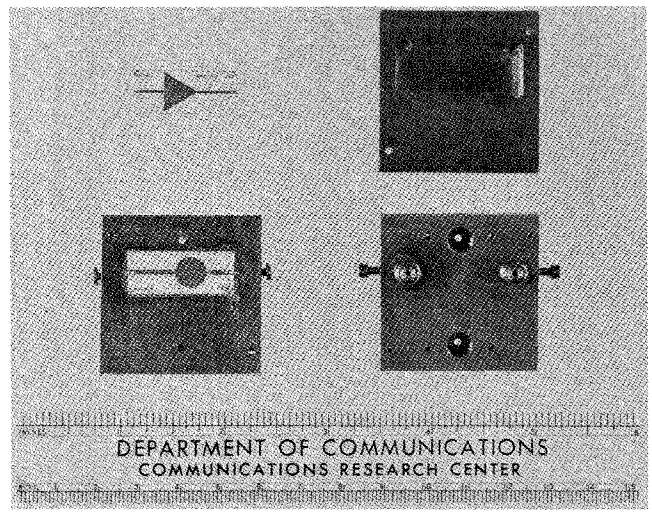
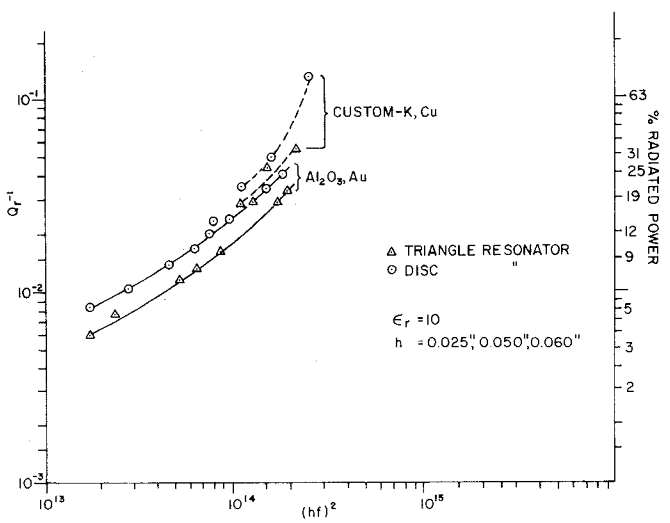
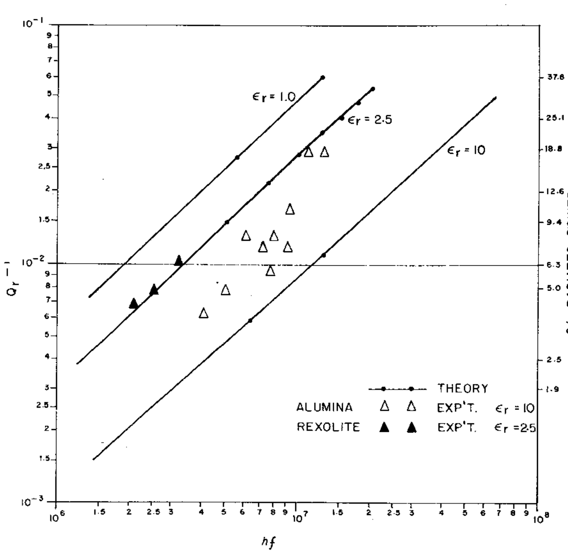


Fig. 3 Photograph of jig used for Q_r measurements



a)



b)

Fig. 4 a) Measured Q_r^{-1} for disc and triangular resonators. b) Measured and theoretical Q_r^{-1} for triangular resonators. } fundamental modes

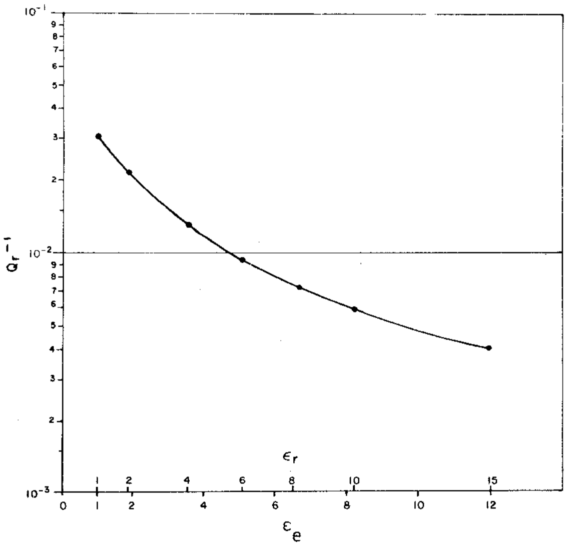
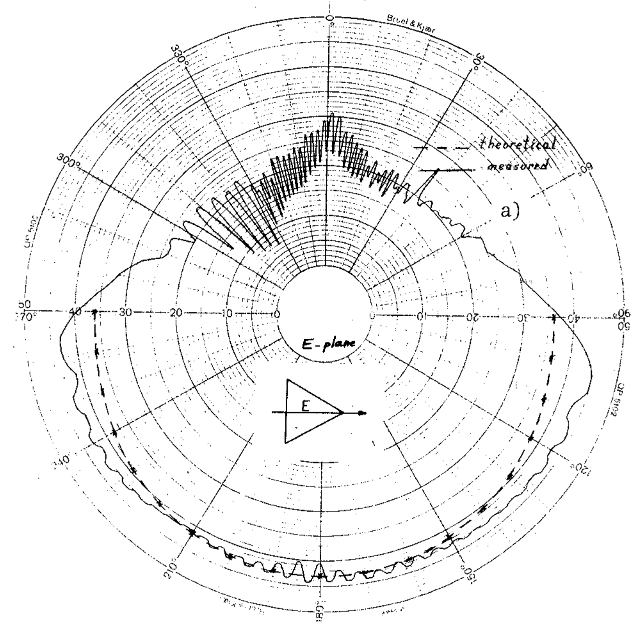


Fig. 5 Theoretical Q_r^{-1} as a function of ε_r; f=10 GHz, h=0.635mm.



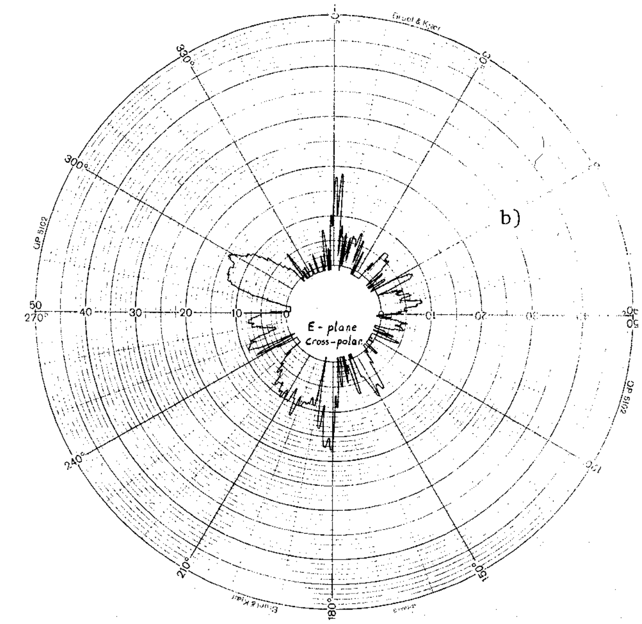


Fig. 6. Measured and theoretical radiation patterns of the fundamental mode in a triangular resonator. f=9.6 GHz, VSWR=1.4, ε_r=10, h=0.635mm.

a) E-plane
b) " cross-polarization
c) H-plane
d) " cross-polarization